METRIC SPACES: FINAL EXAM 2014

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Problem 1. Let $(\mathcal{X}, \mathrm{d})$ be a non-empty metric space, r and s be two positive radii, and $B_r^{\mathrm{d}}(x) = B_s^{\mathrm{d}}(y)$ for some $x, y \in \mathcal{X}$.

- Is it true that $r = s$?
- Is it true that $x = y$?

Problem 2. Let $\mathrm{d}^{(1)}$ and $\mathrm{d}^{(2)}$ be two metrics on a set $\mathcal{X} \neq \varnothing$. By definition, put $\mathrm{d}(x, y) = \max\bigl(\mathrm{d}^{(1)}(x, y), \mathrm{d}^{(2)}(x, y)\bigr)$ for all $x, y \in \mathcal{X}$. Is the function $\mathrm{d}\colon \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ always a metric on $\mathcal{X}$?

Problem 3 (top). Let $A \subseteq \mathcal{X}$ be a connected subset of a space $\mathcal{X}$ and suppose that a non-empty subset $B \subseteq \mathcal{X}$ is such that

$$A \subseteq B \subseteq \overline{A}.$$

Prove that B is connected.

Problem 4 (top). Let A and B be compact subsets of a Hausdorff space $\mathcal{X}$. Prove that their intersection $A \cap B$ is compact.

Problem 5. Prove that the algebraic equation $9x = 1 - x^5$ has a unique solution in the segment $[0, 1] \subset \mathbb{R}$.

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GOOD LUCK!